

## *The attenuation of underwater light*

*by Dr Derek Pilgrim*

THE nature of light, particularly with reference to light under water as well as the physical units used to quantify and measure it, are described in some detail in *The substance of light*, an earlier article in this series. The main points raised were:

- Light is a form of radiant energy. *i.e.* its structure is that of an e-m wave.
- The behaviour of this propagating wave may be described by two entirely compatible models:
  - the wave model, and
  - the photon model.
- The wave model for visible light comprises a range of wavelengths from about 350 nm (deep violet) to 700 nm (deep red).
- The photon is a quantum of light energy,  $e$ , expressed by :

$$e = h\nu = \frac{h.c}{\lambda}$$

(eqn.1)

where  $h$  = Planck's constant =  $6.626 \times 10^{-34}$  Js

$\nu$  = frequency [ $s^{-1}$ ]

$c$  = velocity [ $m.s^{-1}$ ]

$\lambda$  = wavelength [ $m$ ]

- The flux of propagating light is described by a number of terms including:
  - the radiant energy,  $Q(\lambda)$  [J or q = quanta]
  - radiant flux,  $F(\lambda)$  [W,  $qs^{-1}$ , lumen]
  - radiant intensity,  $I(\lambda)$  [ $Wsr^{-1}$ ,  $qs^{-1}sr^{-1}$ , lumen  $sr^{-1}$ ]
  - irradiance,  $E(\lambda)$  [ $Wm^{-2}$ ,  $qs^{-1}m^{-2}$ , lux]. Irradiance may be upwelling, downwelling, scalar or vector

- Light may be measured in  $Watts\ m^{-2}$ ,  $Einstein\ s^{-1}m^{-2}$  or  $lux$  depending upon the use of the measurements. In any particular optical situation only one of these units is correct, and the appropriate metering system should be employed.

The deeper one goes in the ocean, the darker it becomes, indeed, it is apparent that whenever light propagates at any angle through water it loses intensity; that is it *attenuates*. Of course, the degree of attenuation will depend upon the clarity-turbidity of the water.

There are three things that can happen to a photon as it propagates through water:

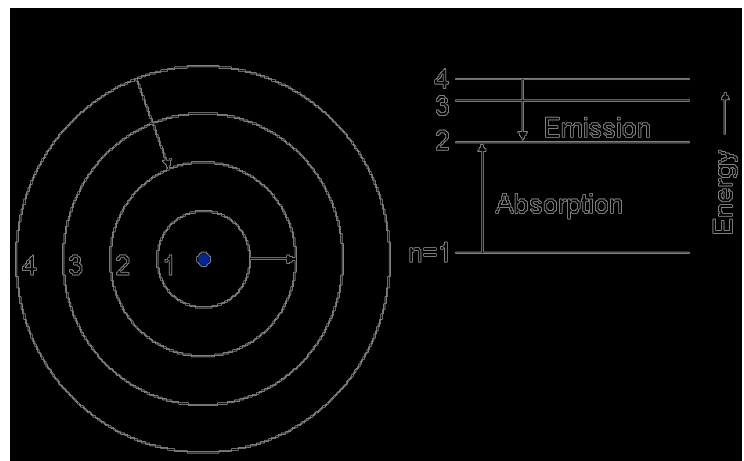
- it can continue undisturbed (*i.e.* continue to be *transmitted*),
- it can be *absorbed*, or
- it can be *scattered*.

When a photon of light energy comes into contact with an atom or molecule of a substance and is absorbed then its energy is transferred to the atom or molecule in question. It is significant that absorption of light is quantised; light energy may be absorbed only in discrete steps or quanta. This is because the energy content of an atom or molecule is limited to a discrete number of levels.

### *Atomic absorption*

The simplest form of absorption is *atomic absorption*. Electrons swarm around a core nucleus, those orbiting furthest from the nucleus possessing more energy than those orbiting closest. According to the quantum theory the electrons can have only certain energy levels so only certain *orbitals* are possible (see *Fig.1*). It therefore follows that the atom can

absorb energy in discrete amounts only. This may be likened to the discrete increases in potential energy that would result from moving a weight from step to step up a staircase, (though in the atom the 'steps' are not evenly spaced). If insufficient energy is available to raise an electron from one orbital to the next then it will remain in its original orbital and no energy will be absorbed. If sufficient energy is available to, say, raise an electron to beyond the next orbital but not to the orbital beyond that, then the electron will move only to the next orbital and one quantum of energy will be absorbed. One may extend the staircase analogy and imagine jumping up a staircase; no matter how large the leap one would necessarily land on a step, not between two steps, and the resulting change in potential energy would be quantised.



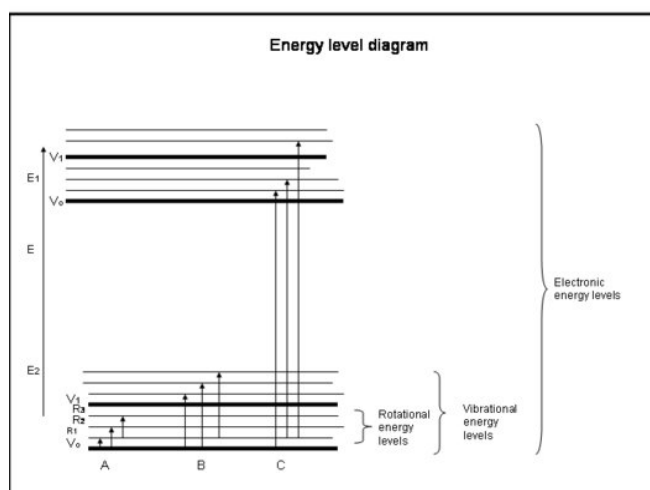
*Fig.1 The quantum nature of light in absorption and emission in an atom*

### *Vibrational and rotational absorption*

Of course, a molecule has a number of electronic energy levels associated with its component atoms; it also has quantised *vibrational* and *rotational* energy levels. Vibrational energy levels are associated with oscillations in atom-atom bond distances—one may imagine atoms joined together by vibrating springs. Rotational energy levels arise from the energy involved in the rotation of molecules in space.

### *Energy level diagrams*

These energy levels are illustrated in the energy diagram in *Fig.2* which shows several vibrational energy levels associated with each electronic energy level and several rotational energy levels associated with each vibrational energy level. As is clear from *Fig.2*, energies associated with the transitions between different rotational energy levels are much smaller than those for vibrations which are, in turn, less than those involved in changes in electronic energy levels. The important point to note here is that the configuration of levels results in a great number of possible *energy transitions*.



*Fig.2 Energy level diagram*

Now, from *eqn.1* the energy in a photon,  $e$ , is expressed by:

$$e = h\nu = \frac{h.c}{\lambda}$$

Since the energy transition between any two levels involves the absorption of a simple photon then it follows that different transitions involve the absorption of photons of different energy content and hence—from *eqn.1*—different frequencies or *colours*. That is to say that the particular configuration of electronic-vibrational-rotational energy levels for any particular molecule *will preferentially absorb particular wavelengths or bands of wavelengths, i.e. colours*.

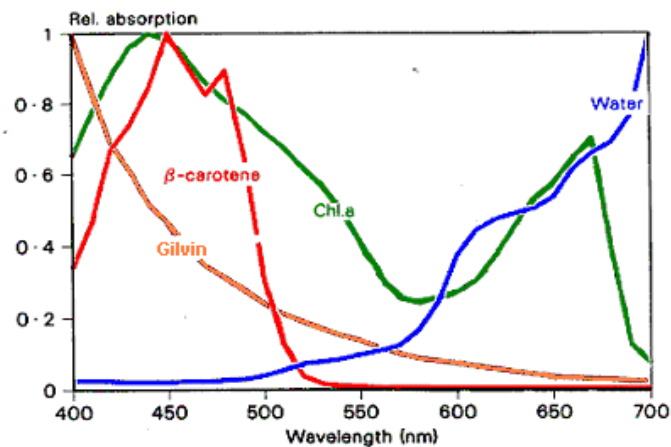
For example, the absorption of visible light by water involves not high energy electronic excitation but lower energy molecular vibration. This is why water selectively absorbs at the low energy, long wave (red) end of the spectrum, (see *Fig.3*); the distinct shoulders in the curve at 515 nm and 604 nm are associated with the sixth and fifth harmonics of O-H

vibration. The low absorption of high energy blue light gives rise, in part, to the characteristic blue colour of water.

Chlorophyll-*a*, on the other hand, has an extended conjugated system of high energy carbon double-bonds (C=C). One of its two absorption bands, centred at 440 and 660 nm, is therefore in the blue; the unutilised band centred at 550 nm is reflected as the characteristic green colour of chlorophyll, (again, see *Fig.3*).

### *Absorption by pigments*

The absorption spectra of water, chlorophyll-*a* and two other important absorbers in the oceans are illustrated in *Fig.3*. In marine algae, the absorption band of chlorophyll-*a* may be extended into the green by the action of accessory pigments, principally: chlorophyll-*b*, -*c*<sub>1</sub>, -*c*<sub>2</sub> and the many carotinoids and biliproteins. For example, the absorption spectrum of *b*-carotene is illustrated in *Fig.3*; absorption is maximum at about 450 nm and zero at  $\lambda > 550$  nm, hence its 'carrot-red' colour.



*Fig.3 Relative absorption spectra of pure water, chlorophyll-a,  $\beta$ -carotene and gilvin, using data from: Smith & Baker (1981) for pure water, Morel & Prieur (1977) for Chlorophyll-a, Kirk (1983) for  $\beta$ -carotene, and Kirk (1976) for typical gilvin*

Natural waters contain yellow substance sometimes known by its German name: Gelbstoff; Kirk (1976) coined the term *gilvin*. Gilvin is produced during the decomposition of organic matter by bacteria. As is apparent from *Fig.3*, gilvin selectively absorbs the high energy, shortwave, blue end of the visible spectrum—hence its characteristic yellow-orange colour. Values of the absorption coefficient,  $a(\lambda)$ , for pure water in the visible spectrum are given in *Table 1* below. Note that absorption is minimum, and scattering is maximum, at the long wave (blue) end of the visible spectrum. This gives rise to the blue colour of water. Since the dissolved salts in sea water have negligible effect upon optical properties, the values tabulated in *Table 3.1* may be taken as valid for the very purest *filtered* seawater.

*Table 1. Absorption coefficients  $a(\lambda)$  for pure water (data from Smith and Baker , 1981)*

$\lambda$	410	470	535	555	575	600	660
colour	violet	blue	green	$V(\lambda)=1$ *	yellow	orange	red
$a(\lambda) m^{-1}$	0.016	0.016	0.053	0.067	0.094	0.244	0.400
$b(\lambda) m^{-1}$	0.007	0.004	0.002	0.002	0.002	0.001	0.001

\* 555 nm is the peak of the human photopic curve i.e. the yellow-green colour to which normal human vision is most sensitive

When a photon interacts with a molecule or particle it may not necessarily be absorbed—it may simply undergo a change in direction. This scattering does not reduce the energy content of a photon, though the resulting increase in pathlength followed by a multi-scattered photon does increase its probability of meeting an absorbing molecule. There are two basic forms of scattering: *density fluctuation scattering* and *particle scattering*. These will now be considered.

### ***Density fluctuation or Rayleigh scattering***

According to the Rayleigh theory, when a particle such as an air molecule is placed in a light field, a dipole is induced by the electric vector of the field. As the dipole oscillates at the frequency of the exciting radiation then it emits radiation of the same frequency in all directions. This re-radiation is the scattered light. The main features of Rayleigh scattering are:

- the scattering is proportional to  $\lambda^{-4}$ , i.e. there is greater scattering at smaller wavelengths—blue in the visible band
- the scattering is proportional to  $d^2$ , where  $d$  is the diameter of the
- molecule, i.e. scattering is greatest for the smallest molecules
- the scattering is symmetrical about the direction of propagation, i.e. the light is scattered in all directions.

Strictly, Rayleigh scattering is applicable only to gasses (the atmosphere, for example) and not to dense fields such as water. However, a type of  $\lambda^{-4}$  molecular scattering caused by the molecules of the water itself and its dissolved salts does occur in very clear seawater and is often referred to loosely as Rayleigh scattering. The scattering coefficient,  $b(m^{-1})$  of water is given for a range of wavelengths in the visible band in *Table 2*. As is to be expected for  $\lambda^{-4}$  type scattering,  $b$  is minimum in the long wave red and maximum in the short wave blue. It is the high back-scattering of blue combined with the high absorption of red that gives clear water its characteristic blue colour, (e.g. see *Fig.3*).

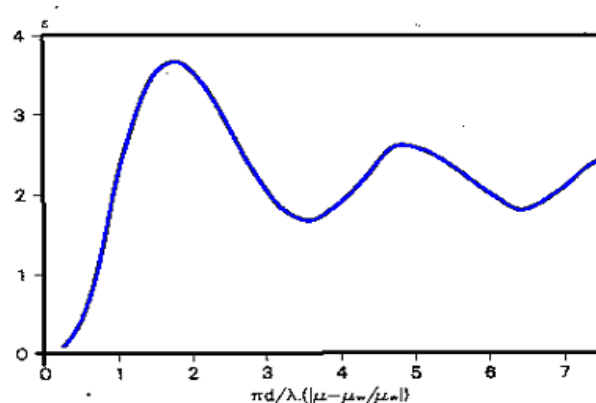
**Table 2 Absorption coefficients  $a(\lambda)$  and scattering coefficients  $b(\lambda)$  for pure water (data from Smith and Baker , 1981)**

$\lambda$	410	470	535	555	575	600	660
colour	violet	blue	green	$V(\lambda)=1^*$	yellow	orange	red
$a(\lambda) m^{-1}$	0.016	0.016	0.053	0.067	0.094	0.244	0.400
$b(\lambda) m^{-1}$	0.007	0.004	0.002	0.002	0.002	0.001	0.001

\* 555 nm is the peak of the human photopic curve i.e. the yellow-green colour to which normal human vision is most sensitive

### Particle (Mie) scattering

Whilst molecular scatterers are small relative to the wavelength of light, a number of workers, (see Jerlov, 1968, for summary), have shown that most of the scattering of light in natural waters is from particles of diameter,  $d > 2\mu m$ , i.e. particles large compared to the wavelength of light. Scattering by these large particles comprises three phenomena: diffraction, reflection (internal and external) and refraction. The diffraction is determined by the size and shape of the particle, whereas refraction and reflection depends upon its composition (refractive index). The basis for predicting the light scattering behaviour of spherical particles of any size is the Mie theory, developed by Mie in 1908. The great advantage of the Mie theory is that it is all-embracing; for very small particles, for example, it leads to the same predictions as the Rayleigh theory. However, the analytical expressions of the Mie theory are very complex and are best dealt with by computer. Certainly the complex Mie theory is beyond the scope of this article, but its result is illustrated in Fig.4, where  $\xi$  is the scattering efficiency, (effective scattering area over real area),  $d$  is the particle diameter,  $\lambda$  is the wavelength of the scattered light and  $\mu, \mu_w$ , are the refractive indices of the particle and water. It is apparent that when the relative particle size,  $d/\lambda$  is small then the scattering is small but rises to a maximum as particle size increases. With further increase in  $d/\lambda$ , the value of  $\xi$  decreases, oscillates and eventually settles about a value of  $\xi = 2$ , at which point it becomes independent of  $\lambda$ , i.e. all colours are scattered equally. Unlike the symmetrical Rayleigh scattering, particle scattering is predominantly in the general direction of propagation, i.e. forward rather than backward.



**Fig.4 Mie scattering**

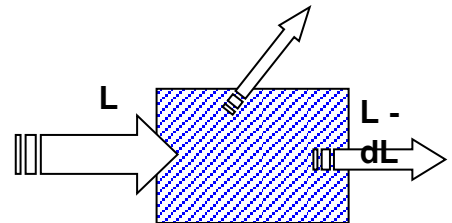
## Scattering coefficient of water

The values of the scattering coefficient,  $b(\lambda)$  for pure water in the visible band are given in *Table 2* above. Note that scattering is maximum at the long wave (blue) end of the visible spectrum. This contributes to the blue colour of water. Since the dissolved salts in sea water have negligible effect upon optical properties, the values tabulated in *Table 2* may be taken as valid for the very purest *filtered* seawater. We may now consider the *combined* effects of the absorption and scattering phenomena in reducing a flux of photons in an absorbing-scattering medium such as water.

## Attenuance, $A$ , and transmittance, $T$

Suppose that we have a narrow beam of collimated light, that is radiance,  $L$ , propagating through a short distance,  $dr$ , of water (see *Fig.5*). Suppose also that over this distance there is an intensity loss of  $dL$ , so that the incident intensity is attenuated to  $L - dL$ . We may therefore write:

$$T = \frac{L - dL}{L} = 1 - \frac{dL}{L} \quad (\text{eqn.2})$$



*Fig.5 The attenuation of radiance*

where  $T$  is the *beam transmittance*, that is the ratio of "what comes out divided by what goes in", and the loss  $-dL/L$  is the *attenuance*,  $A$ .

## The beam attenuation coefficient, $c$

It is apparent that the attenuance,  $A$ , does not fully describe the optical turbidity of the water since it does not specify over what *distance* of water the radiance is attenuated by  $dL/L$ . We therefore introduce the *beam attenuation coefficient*,  $c$ , defined as the attenuance of an infinitely thin layer of water divided by the layer thickness:

$$c = \frac{A}{dr} = -\frac{dL/L}{dr} = -\frac{1}{L} \cdot \frac{dL}{dr} = -\frac{d}{dr} \ln[L] \quad (\text{eqn.3})$$

*i.e.*  $c$  is the rate of change, with distance, of the natural logarithm of the radiance intensity. Note that  $dL/dr$  is negative, but that the beam attenuation coefficient,  $c$ , is always positive, (a negative  $c$  would imply an increase in light intensity with distance). Note also that the units of  $c$  are  $\text{m}^{-1}$  (per meter). In the early hydrooptical literature, the beam attenuation coefficient was indicated by  $\alpha$  (alpha); you may still come across this usage.

### ***Beer Lambert's law***

This law fully describes the logarithmic attenuation of underwater light, and is widely used in the study of underwater optics. For radiance, it is derived as follows. We have eqn.3:

$$c = - \frac{d}{dr} \ln[L]$$

This equation describes attenuation over the small distance  $dr$  only. We need to consider attenuation over the realistic distance  $r$ . So, integrating both sides of eqn.3 between 0 and  $r$ :

$$\int_0^r c \cdot dr = - \int_0^r \frac{d}{dr} \ln[L] \cdot dr$$

$$\therefore -c \int_0^r dr = \int_0^r d \ln[L]$$

$$\therefore -c[r]_0^r = [\ln L]_0^r$$

$$\therefore -c \cdot r = \ln[L(r)] - \ln[L(0)]$$

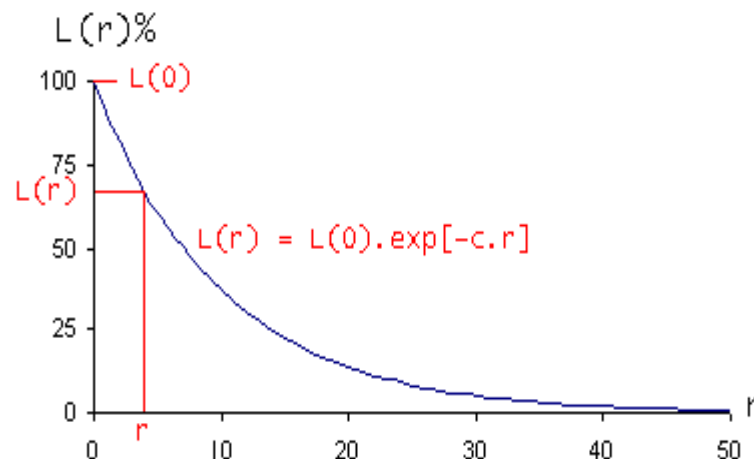
$$\therefore \frac{L(r)}{L(0)} = T = e^{-c \cdot r} = \exp[-c \cdot r]$$

$$\therefore L(r) = L(0) \cdot \exp[-c \cdot r]$$

***Beer-Lambert's Law for radiance***

(eqn.4)

Of course Beer-Lambert's law represents *exponential decay* as illustrated in Fig.6.



*Fig. 6 Beer-Lambert's law for radiance*



It is a well known feature of exponential decay that the quantity involved gets smaller and smaller but never becomes zero. For example, in Beer-Lambert's law (eqn.4) at what distance,  $r$ , would radiance,  $L(r)$ , become zero?

$$0 = L(0) \cdot \exp[-c \cdot r]$$

$$\therefore 0 = \exp[-cr]$$

$$\therefore r = \infty$$

*i.e.*  $L(r)$  would never become zero no matter how far it travelled. Of course, this is not true in practice—and it's not true in theory either! We know that light travels as photons which are *quanta*, that is the *smallest* possible quantities of light, (see the article *The substance of light* in this series). Once a light flux is reduced to just one photon then the next reduction must be to zero photons—reduction to a fraction of a photon is clearly not possible.

### *The optical length/distance, $\gamma$*

The exponent  $cr$  in Beer-Lambert's law, that is the geometrical distance times the beam attenuation coefficient, is termed the *optical length*,  $\gamma$ , *i.e.*:

$$\gamma = c \cdot r \tag{eqn.5}$$

A given optical length of water will result in a particular attenuation. For example, light attenuates equally when propagating through 5 m of water of  $c = 0.3 \text{ m}^{-1}$ , and when propagating through 10 m of water of  $c = 0.15 \text{ m}^{-1}$ . In both cases the optical length,  $\gamma$ , is  $5 \times 0.3 = 10 \times 0.15 = 1.5$ , and the beam transmittance,  $T$ , is given by:

$$\begin{aligned} T &= \frac{L(r)}{L(0)} = \exp[-cr] = \exp[-\gamma] \\ &= \exp[-1.5] = 22.3\% \end{aligned}$$

Similarly, we may expect to see a black object against the water background at an optical distance of 4.6, (*e.g.* at a distance of 4.6 m in water of  $c = 1.0 \text{ m}^{-1}$ , and at a distance of 20 m in water of  $c = 0.23 \text{ m}^{-1}$  and so on).

Note that  $\gamma$ , is dimensionless [ $\text{m} \times \text{m}^{-1}$ ] so is not, strictly, a 'length' at all.

### *The attenuation length, $\ell$*

The beam attenuation coefficient,  $c \text{ [m}^{-1}\text{]}$ , may not be easy to visualise. Some optical oceanographers recognise this and prefer to use its reciprocal, the *attenuation length*,  $\ell \text{ [m]}$  as it is a more easily handled *length*, *i.e.*:

$$\ell(\text{m}) = \frac{1}{c(\text{m}^{-1})}; c(\text{m}^{-1}) = \frac{1}{\ell(\text{m})} \tag{eqn.6}$$

So, from Beer-Lambert's law (eqn.4):

$$L(r) = L(0) \cdot \exp[-cr] = L(0) \cdot \exp\left[-\frac{r}{\ell}\right] \quad (\text{eqn.7})$$

But what is  $\ell$ ? What is this *length*? Suppose that in a particular situation described by Beer-Lambert's law:

$r = \ell$ ; so from eqn.7:

$$\begin{aligned} \frac{L(r)}{L(0)} &= \exp[-cr] = \exp\left[-\frac{r}{\ell}\right] = \exp\left[-\frac{\ell}{\ell}\right] \\ &= \exp[-1] = 0.3679 \end{aligned}$$

So,  $\ell$  is the distance over which the radiance is reduced to 36.8% of its starting value. Over a distance of  $2\ell$ , the radiance is reduced to  $36.8\% \times 36.8\% = 13.5\%$  of its starting value, and so on. This is illustrated in Fig.7, below.

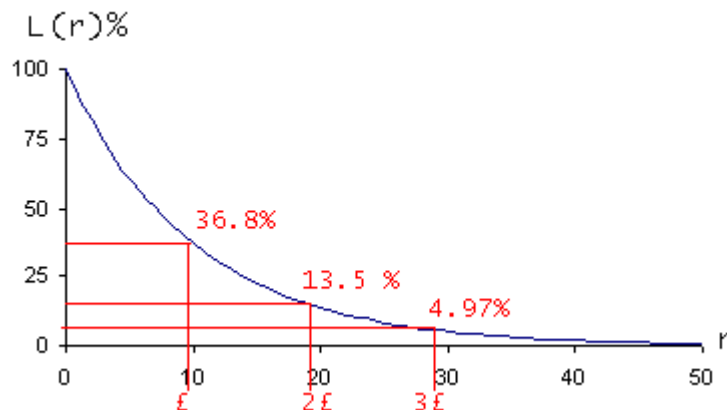


Fig.7 Beer-Lambert's law in terms of the attenuation length,  $\ell$

### ***The half-distance, $\omega$***

Although the attenuation length,  $\ell$ , is more easily visualised than the beam attenuation coefficient,  $c$ , it is a pity that it gives rise to an attenuation of 36.8% and not, say, 50% as in the *half life* concept of radio activity, electronics, etc. Pilgrim (1999) introduced a convenient half distance,  $\omega$ , over which radiance is reduced to 50% of its starting value. What, then, is the relationship between  $\omega$  and the beam attenuation coefficient,  $c$ ?

$$\text{let } c \cdot \omega = n, \text{ i.e. } c = \frac{n}{\omega}$$

So, in Beer-Lambert's law (Eqn.4):

$$L(r) = L(0). \exp[-c.r] = L(0). \exp\left[-\frac{n}{\omega}.r\right]$$

By definition of  $\omega$ , when  $r = \omega$ ,  $L(r) = \frac{1}{2} L(0). \exp[-n]$  so that:

$$\frac{1}{2} L(0) = L(0). \exp\left[-\frac{n}{\omega}. \omega\right] = L(0). \exp[-n]$$

$$\therefore n = -\ln\left[\frac{1}{2}\right] = \ln[2] = 0.6931 \approx 0.7$$

$$\therefore c. \omega = \ln[2] \approx 0.7$$

$$\text{i.e. } \omega = \frac{0.7}{c}$$

(eqn.8)

and so we may write:

$$L(r) = L(0). \exp[-c.r] = L(0). \exp\left[-\frac{0.7r}{\omega}\right]$$

(eqn.9)

This is illustrated in Fig.8.

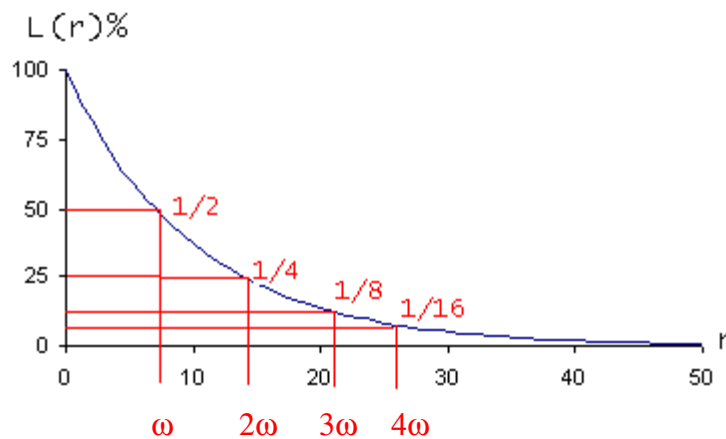


Fig.8 Beer-Lambert's law in terms of the half distance,  $\omega$

Values of  $\omega(\lambda)$  for pure water at a selection of wavelengths in the visible band are presented in Table.3 on the following page Note from the table that whilst a red beam of radiance would be reduced to half intensity by less than 2m of clear water, blue radiance would be reduced to half only after propagating through over 30m of water.

Note that absorption is minimum, and scattering is maximum, at the long wave (blue) end of the visible spectrum. This gives rise to the blue colour of water. Since the dissolved salts in sea water have negligible effect upon optical properties, the values tabulated in Table.3 may be taken as valid for the very purest filtered seawater.

**Table 3** *Absorption, scattering, attenuation and diffuse attenuation coefficients for pure water (data from Smith and Baker , 1981)*

$\lambda$	colour	$a(m^{-1})$	$b(m^{-1})$	$c(m^{-1})$	$\omega(m)$	$K(m^{-1})$	$\omega_K(m)$
410	violet	0.016	0.007	0.023	30.14	0.020	34.66
470	blue	0.016	0.004	0.020	34.66	0.018	38.51
535	green	0.053	0.002	0.055	12.60	0.054	12.84
555	$V(\lambda)=1^*$	0.067	0.002	0.069	10.05	0.068	10.19
575	yellow	0.094	0.002	0.096	7.22	0.095	7.30
600	orange	0.244	0.001	0.245	2.83	0.245	2.83
660	red	0.400	0.001	0.401	1.73	0.400	1.73

\* 555 nm is the peak of the human photopic curve i.e. the yellow-green colour to which normal human vision is most sensitive

### ***The beam transmissometer***

The beam transmittance,  $T$ , may be measured *in situ* by means of an instrument known as a *beam transmissometer*. The beam attenuation coefficient,  $c$ , is then calculated from  $T$ . The beam transmissometer is described and explained in detail in the associated article *Measurement of the attenuation of underwater light*.

### ***The diffuse attenuation coefficients, $K$ , $K_u$ , $K_d$***

Thus far we have considered the attenuation of collimated radiance,  $L$ , only. Of course a field of diffuse irradiance,  $E$ , is also subject to attenuation—*diffuse attenuation*—which may also be described by Beer-Lambert's law. Usually, we associate decreasing irradiance specifically with depth,  $z$ , so that the most common form of this equation is:

$$E(z) = E(0) \cdot \exp[-K \cdot z]$$

***Beer-Lambert's Law for irradiance***

(eqn.10)

where  $K$  is the *diffuse attenuation coefficient*—sometimes called the *extinction coefficient*.

By simple modification of eqn.3:

$$K(m^{-1}) = -\frac{1}{E} \cdot \frac{dE}{dz} = -\frac{d}{dz} \ln[E] \quad (\text{eqn.11})$$

We may specify irradiance as *downwelling irradiance*,  $E_d$ , or *upwelling irradiance*,  $E_u$ . Associated with these fluxes are the *downwelling diffuse attenuation coefficient*,  $K_d$ , and the *upwelling diffuse attenuation coefficient*,  $K_u$ :

$$\begin{aligned} K_d &= -\frac{1}{E_d} \cdot \frac{dE_d}{dz} = -\frac{d}{dz} \ln[E_d] \\ K_u &= -\frac{1}{E_u} \cdot \frac{dE_u}{dz} = -\frac{d}{dz} \ln[E_u] \end{aligned} \quad (\text{eqn.12})$$

It is apparent that although the flux of irradiance may become attenuated by absorption, just as in the case of collimated light, scattered-only photons are not lost, but follow a longer path length. The longer path length is, of course, due to scattering, and gives rise to an *increased chance of absorption*. This scattering-dependent additional absorption is commonly described by the term:  $b/n$ , *i.e.*:

$$K = a + \frac{b}{n} \quad (\text{eqn.13})$$

Sørensen *et al* (1975) have inspected a variety of measured data from different sources, and have suggested that  $n \sim 6$ , *i.e.*:

$$K = a + \frac{b}{6} \quad (\text{eqn.14})$$

The most commonly considered form of  $K$  is  $K_d$ , the downwelling diffuse attenuation coefficient and Kirk (1981) has found, by Monte Carlo modelling, the relationship:

$$K_d = (a^2 + 0.256ab)^{\frac{1}{2}} \quad (\text{eqn.15})$$

Preisendorfer (1961) expressed the structure of this coefficient as:

$$K_d = a_d + b_{bd} - b_{bu} \cdot R \quad (\text{eqn.16})$$

where  $a_d$  is the absorption coefficient of the downwelling irradiance,  $b_d$  is the back, *i.e.* upward, scattering coefficient of the downwelling irradiance,  $b_{bu}$  is the back, *i.e.* downward, scattering coefficient of the *upwelling* irradiance, and  $R$  is the irradiance reflectance. Values of  $K(\lambda)$  for a selection of wavelengths in the visible band were presented in *Table.3*.

***The diffuse optical length,  $\gamma_K$ , diffuse attenuation length,  $\mathcal{L}_K$ , and diffuse half-distance,  $\omega_K$***

Equivalent to the optical length/depth,  $\gamma$ , attenuation length/depth,  $\mathcal{L}$ , and half distance,  $\omega$ , for

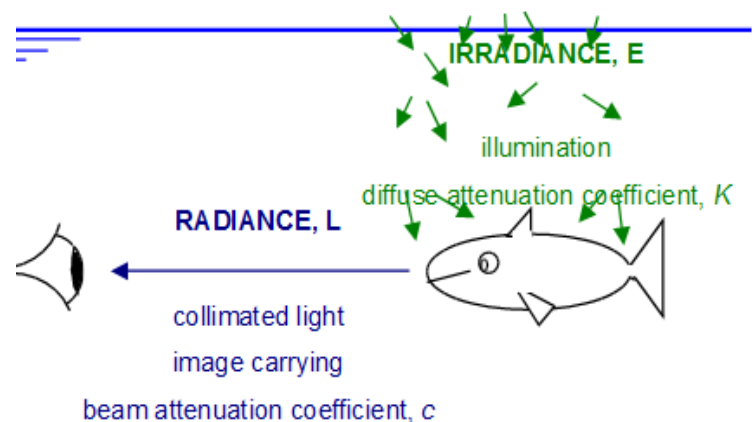
collimated light (eqns.5, 6 and 8), are the diffuse optical length/depth,  $\gamma_K$ , diffuse attenuation length/depth,  $\ell_K$ , and diffuse half-distance  $\omega_K$ , for diffuse radiance:

$$\gamma_K = K.z; \quad J_K = \frac{1}{K}; \quad \omega_K = \frac{0.7}{K} \quad (\text{eqn.17})$$

Values of  $\omega_K(\lambda)$  for a selection of wavelengths in the visible band are presented in *Table.3* on page 12. Again, compare the poor transmission of irradiance at long wavelengths (red end of the visible spectrum) with that at short wavelengths (blue end).

### The ratio $c/K$

The beam attenuation coefficient,  $c$ , governs the propagation of collimated light and is, therefore, associated with problems of imaging (visibility, photography etc). The diffuse attenuation coefficient,  $K$ , governs the propagation of diffuse light and is thus associated with illumination. This relationship is illustrated in *Fig.10*.



*Fig.10 Relationship between radiance and irradiance*

We have, (eqns.11 and 17):

$$c = a + b$$

$$K = a + \frac{b}{6}$$

As is apparent from *Table 1*, in pure seawater  $c$  is only slightly bigger than  $K$  at all wavelengths. With increase in turbidity there is greater scattering and the scattering coefficient,  $b$ , associated with  $c$ , increases at 6 times the rate of  $b/6$ , the additional absorption associated with  $K$ . Of course, this is because photons scattered out of a collimated beam are lost from the beam, whilst photons scattered within a field of diffuse light are not necessarily lost—they only suffer a *greater chance of loss by absorption because they are made to follow longer paths*. A classic consequence of this is that a small increase in turbidity results in a significant decrease in the range of visibility but gives rise to only a small decrease in the general light level, (the same phenomenon is observed in fog). So, with increase in turbidity,  $c$  increases more than  $K$ ; specifically the ratio  $c/K$  increases. The  $c/K$  ratio is therefore a very good indicator of scattering within the water. In pure seawater it is a little over 1; in fairly clear deep ocean water it is about 2.7 (Tyler, 1968); in turbid estuarine water it can be 10 or more (Pilgrim, 1988). Shannon (1975) measured a range of values of  $c$  ( $0.1 \leq c \leq 1.6$ ) and  $K$  off the east coast of the USA and Caribbean and found, as a first order approximation, ( $r^2 = 0.96$ ):

$$K = 0.2c + 0.04; \quad c = 5(K - 0.04) \quad (\text{eqn.18})$$

At the measured limits of optical turbidity this equation indicates the ratio:  $c/K = 1.77$  at  $c = 0.11 \text{ m}^{-1}$ , and  $c/K = 4.44$  at  $c = 1.6 \text{ m}^{-1}$ .

### *The $\ln[E(z)]$ and $\ln[L(r)]$ curves*

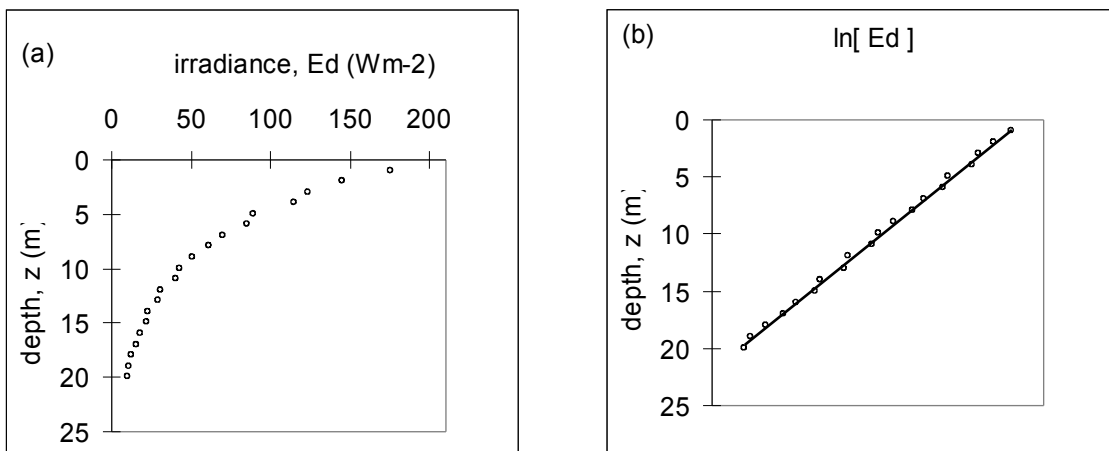
We have Beer-Lambert's law for irradiance, (eqn.13):

$$E(z) = E(0) \cdot \exp[-K \cdot z]$$

and taking logs throughout:

$$\ln[E(z)] = -K \cdot z + \ln[E(0)]$$

which is the  $y = \text{gradient} \times x + \text{intercept}$  form of a linear equation, as illustrated in Fig.11 below. Note that  $-K$  is given by the gradient of the  $\ln[E(z)]$  curve. This was deduced earlier in eqn.14:



*Fig.10 Depth profiles of  $E(z)$  and equivalent  $\ln[E(z)]$*

Of course, a similar linear plot of  $\ln[L(r)]$  may be produced, in which case the gradient is:

$$c = -d/dr \{ \ln[L(r)] \}$$

The construction of  $\ln[E(z)]$  or  $\ln[L(r)]$  plots provide a useful method of determining  $K$  or  $c$  from observed profiles of  $E(z)$  or  $L(r)$ . This technique is examined in further detail in the article [Measuring the attenuation of underwater light](#) in this series

### *Scalar diffuse attenuation coefficients*

Of course, associated with scalar irradiances  $E_0$ ,  $E_{od}$  and  $E_{ou}$  etc are scalar diffuse attenuation coefficients  $K_0$ ,  $K_{od}$ ,  $K_{ou}$  etc.

### *Ocean colour*

We see from Table 3 that clearest ocean water absorbs different wavelengths (colours) at different rates. This is the fundamental cause of 'ocean colour', a topic dealt with in detail in the article [Ocean colour](#) in this series.

## Summary

- Light is attenuated under water by a combination of absorption (atomic, vibrational and rotational ) and scattering (density fluctuation—Rayleigh, or particle—Mie)
- Vibrational absorption of H<sub>2</sub>O and Mie scattering by H<sub>2</sub>O molecule give rise to the pale blue colour of clear water – including sea water.
- Absorption by various pigments in natural water gives rise to variations in water colour.

Attenuation is described by Beer-Lambert's law for radiance:

$$\therefore L(r) = L(0). \exp[-c.r]$$

and for irradiance:

$$E(z) = E(0). \exp[-K.z]$$

- A number of optical terms may be derived from Beer-Lambert's law including:

the optical distance,  $\gamma$ :

$$\gamma = c.r$$

the attenuation length,  $\ell$ :

$$\ell(\text{m}) = \frac{1}{c(\text{m}^{-1})}; c(\text{m}^{-1}) = \frac{1}{\ell(\text{m})}$$

and the half distance,  $\omega$ :

$$\text{i.e. } \omega = \frac{0.7}{c}$$

- Diffuse irradiation,  $E$ , is also subject to attenuation—diffuse attenuation—which may be described by Beer-Lambert's law:

$$E(z) = E(0). \exp[-K.z]$$

and this gives rise to a number of optical terms including the diffuse optical length/depth,  $\gamma_K$ , diffuse attenuation length/depth,  $\ell_K$ , and diffuse half-distance  $\omega_K$ , for diffuse radiance:

$$\gamma_K = K.z; \quad \ell_K = \frac{1}{K}; \quad \omega_K = \frac{0.7}{K}$$



- A small increase in turbidity results in a comparatively large increase in the ratio  $c/K$  and a significant decrease in the range of visibility, but to only a small decrease in the general level of illumination.
- The gradients of the  $\ln[E(z)]$  and  $\ln[L(r)]$  curves provide a means of deriving the diffuse and beam attenuation coefficients,  $K$  and  $c$  respectively:

$$K = -d/dz\{\ln[E(z)]\}$$

$$c = -d/dr \{\ln[L(r)]\}$$

## *References*

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