The underwater light field

by Dr Derek Pilgrim

The light field describes the *flux* of propagating light. In the definitions following, angles will be expressed in radians. Directions will be specified by the zenith angle, θ, (θ = 0 = zenith; θ = π = nadir), and the azimuth, φ , (0 ≥ φ ≥ 2□). The *units* of the quantities involved are given in square parentheses, [ ].

Radiant energy, Q(λ) [J or q = quanta]

This is the radiant energy that moves in the radiation field, indeed, the whole physical purpose of radiant propagation is to move energy from one place to another.

Radiant flux, F(λ)[W, qs^{-1}, lumen]

Since radiation must be propagating at velocity, *c*, then flux of radiant energy (*i.e.* radiant power), is the fundamental quantity used to describe a radiation field. It is the time rate (d/dt) flow of radiant energy *Q(λ),* and so:

$$F(\lambda) = \frac{\mathrm{d}Q(\lambda)}{\mathrm{d}t} \qquad \textit{(eqn.1)}$$

The photopic (visual) unit of radiant or luminous flux is the lumen, a measure of the power of light perceived by the human eye. It differs from radiant flux, *F*, in that luminous flux takes into account the varying sensitivity of the human eye to different wavelengths of light.

Radiant intensity, I(λ) [Wsr^{-1}, qs^{-1}sr^{-1}, lumen sr^{-1}]

Radiant intensity is the measure of the radiant flux per unit solid angle, ω, (per steradian, sr^{-1}). Consider a radiance source radiating a radiant flux of *F(λ)* watts as in *Fig.1.* If this flux is concentrated into a cone of solid angle 1 steradian, then it has an intensity of *I* W sr^{-1}. We may therefore write:

$$I = \frac{dF(\lambda)}{d\omega} = \frac{d^2Q}{d\omega\,.dt} \qquad \textit{(eqn.2)}$$

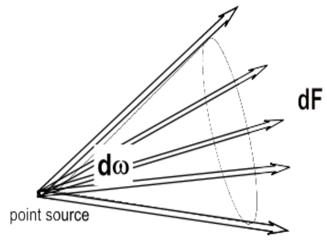

Fig.1 Radiant intensity, I(λ)

Radiance, L(λ) [$Wsr^{-1}m^{-2}$, $qs^{-1}sr^{-1}m^{-2}$, lux]

Suppose that we now replace the radiant source in *Fig.1* with a radiance detector of infinitesimal size. If the field of view of the detector is restricted by a conical shield to an angle of 1 steradian then it will measure radiant flux in units of watts per steradian [Wsr^{-1}]. However, if the detector has finite area, d*A*, and is placed at the end of a tube (Gershun tube) of length *l* as in *Fig.2*, then the field of view of the detector will become restricted to a solid angle of $d\Omega = dA/l^2$, so that the radiant flux (power) arriving at the detector will be: dF(λ)/dΩ.

Fig.2 Radiance, L(λ)

Since the detector has finite area, dA, then the intensity that it measures will be the radiance, L(λ), *i.e*:

$$L(\lambda) = \frac{d^2F(\lambda)}{d\Omega\,.dA} \quad W\,sr^{-1}m^{-2} \qquad \textit{(eqn.3)}$$

Strictly, in defining radiance, $d\Omega \rightarrow 0$, *i.e*. the tube will be infinitely long so that the detected light will comprise parallel beams, *i.e.* it will be collimated light. In practice the tube, or its optical equivalent, need only be so long that a further increase in length would make very little difference to the measured value of L(**λ**). From *eqns.4* and *5* we may write :

$$L(\lambda) = \frac{\mathrm{d}^2 F(\lambda)}{\mathrm{d}A.\mathrm{d}\Omega} = \frac{\mathrm{d}I(\lambda)}{\mathrm{d}A} \qquad \textit{(eqn.4)}$$

***Irr**adiance, E(**λ**)*[Wm^{-2}, $qs^{-1}m^{-2}$, lux]

Simply, the irradiance, *E(**λ**)*, is the radiant flux incident on a surface divided by the area of that element :

$$E(\lambda) = \frac{\mathrm{d}F(\lambda)}{\mathrm{d}A} \qquad \textit{(eqn.5)}$$

This is illustrated in *Fig.3(a)*:

Fig.3 Vector (cosine) irradiance collectors

There are a number of forms of irradiance commonly used in measuring the underwater light field, defined as follows :

Downwelling vector irradiance, Ed(λ), the flux incident per unit area measured on a horizontal, upward facing light collector (cosine collector) as illustrated in *Fig.3(b)*.

Irradiance may be described mathematically as the integration of radiance over a given solid angle, *dω,* so :

$$Ed(\lambda) = \int_{\theta=\pi}^{\theta=-\pi} \int_{\phi=0}^{\phi=2\pi} L(\lambda).\cos\theta\,.\mathrm{d}\omega \qquad \textit{(eqn.6)}$$

The cosine term in e*qn.5* arises because the light arrives at the (cosine) collector at an angle. Consider the beam of light of radiant flux, *F*, in *Fig.3(c)*. Measured at right angles to the beam, the irradiance would be : $\mathrm{d}E_1 = \mathrm{d}F/\mathrm{d}A_1$. However, the irradiance measured by the collector is:

$dE_2 = dF/dA_2$, and since $dA_1 = dA_2.\cos\theta$ then $dE_2 = dE_1.\cos\theta$

It is for this reason that the collector is called a *cosine* collector, and that the measured irradiance is termed *vector* irradiance.

Upwelling vector irradiance, Eu(λ), the equivalent upward flux of irradiance as illustrated in *Fig.3(d)*:

$$Eu(\lambda) = \int_{\theta=-\pi}^{\theta=\pi} \int_{\phi=0}^{\phi=2\pi} L(\lambda).|\cos\theta|.\mathrm{d}\omega \qquad \textit{(eqn.7)}$$

The total scalar irradiance, Eo(λ), the integral of a radiance distribution at a point over all directions about a point and is equivalent to the *total* energy flux, d*F*/d*t*:

$$Eod(\lambda) = \int_{4\pi} L(\lambda).\mathrm{d}\omega \qquad \textit{(eqn.8)}$$

Total scalar irradiance is illustrated in *Fig.3(e)*. Of course, we may also have ***upwelling scalar irradiance, Eou(λ)***and ***downwelling scalar irradiance, Eod(λ)***

$$Eou(\lambda) = \int_{-2\pi} L(\lambda).\mathrm{d}\omega \;\; ; \;\; Eod(\lambda) = \int_{2\pi} L(\lambda).\mathrm{d}\omega \qquad \textit{(eqn.9)}$$

Summary

The flux of propagating light is described by a number of terms including:

- the radiant energy, $Q(\lambda)$ [J or q = quanta]
- radiant flux, $F(\lambda)$ [W, qs^{-1}, lumen]
- radiant intensity, $I(\lambda)$ [Wsr^{-1}, $qs^{-1}sr^{-1}$, lumen sr^{-1}]
- irradiance, E(λ) [Wm-2, qs-1m-2, lux]. Irradiance may be upwelling, downwelling, scalar or vector